Comment on "Gauge Invariance and k_T-Factorization of Exclusive Processes"

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We point out mistakes made in the one-loop calculation of some diagrams for the process $\pi\gamma^* \to \gamma$ in the preprint arXiv:0807.0296, and present correct results. Especially, we have difficulty to understand their argument that a highly off-shell gluon generates a light-cone (infrared) singularity. It is shown by means of the Ward identity that the gauge-dependent light-cone singularity found in arXiv:0807.0296 does not exist. It is then shown that a hard kernel derived in the k_T factorization of exclusive processes is gauge invariant and free of the light-cone singularity.

In a recent preprint arXiv:0807.0296 [1], the authors studied the pion transition form factor in the k_T factorization theorem at one-loop level, calculating the diagrams in full QCD for the form factor (see their Fig. 4) and the diagrams for the k_T-dependent pion wave function (see their Figs. 2 and 3) in the covariant gauge. They focused on a special piece of contribution, which depends on the gauge parameter and contains a light-cone singularity. It was found that the full QCD diagrams do not generate this contribution, but those for the pion wave function do, if the involved partons are taken to be off-shell. Therefore, the hard kernel, defined as the difference of the above two sets of diagrams, is gauge dependent, and contains the light-cone singularity. They were then led to the conclusion that the k_T factorization theorem for exclusive processes violates gauge invariance, and that the perturbative QCD (PQCD) approach [2, 3, 4, 5] to exclusive B meson decays, based on the k_T factorization theorem, is also gauge dependent. This contradicts the conclusions drawn in our previous work [6]. In this comment we point out the mistakes made in their calculation, and demonstrate that the gauge-dependent light-cone singularity discussed by the authors of [1] does not exist. The all-order proof for the gauge invariance and infrared finiteness of a hard kernel derived in the k_T factorization [6] is indeed correct.

Consider the process $\pi(P)\gamma^* \to \gamma(p)$ in Fig. 1, with P (p) being the pion (outgoing on-shell photon) momentum along the plus (minus) direction. Define the momenta of the valence quark and anti-quark in the pion as [1]

$$k_1^\mu = (k_1^+, 0, \vec{k}_{1\perp}) \, , \qquad k_2^\mu = (k_2^+, 0, -\vec{k}_{1\perp}) \, , \tag{1}$$

respectively, with $k_1^+ = x_0 P^+$ and $k_2^+ = (1-x_0)P^+$, x_0 being the momentum fraction. The leading-order (LO) hard kernel is given, in terms of the above momenta, by

$$H^{(0)} = \frac{1}{2k_1^+ p^- + k_{1\perp}^2} = \frac{1}{x_0 Q^2 + k_{1\perp}^2} \, , \tag{2}$$

with the momentum transfer $Q^2 = 2P^+p^-$. At next-to-leading order (NLO), a loop momentum q carried by the additional gluon may flow through the hard kernel, for example, in Fig. 2. In this case the factorized k_T-dependent wave function is convoluted with

$$H^{(0)} = \frac{1}{2(k_1^+ - q^+)p^- + |\vec{k}_{1\perp} - \vec{q}_\perp|^2} \, . \tag{3}$$

We find that the mistakes made in the calculation of Fig. 2 and Fig. 3 [1] arise from an improper application of the contour integration. Take the simple loop integral associated with Figs. 2(d), 3(b) and 3(e) as an example:

$$I = 16i\alpha g_s^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - \lambda_L^2 + i\varepsilon)[q^2 - (1-\alpha)\lambda_L^2 + i\varepsilon]} \, , \tag{4}$$

where the gauge parameter α comes from the gluon propagator in the covariant gauge [1]

$$\frac{-i}{q^2 - \lambda_L^2 + i\varepsilon}\left[g^{\mu\nu} - \alpha \frac{q^\mu q^\nu}{q^2 - (1-\alpha)\lambda_L^2 + i\varepsilon}\right] \, , \tag{5}$$

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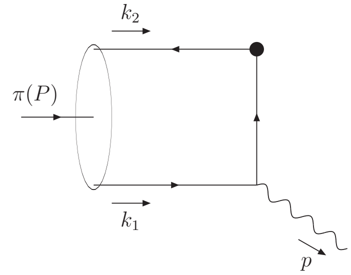

FIG. 1: LO diagram for $\pi(P)\gamma^* \to \gamma(p)$.

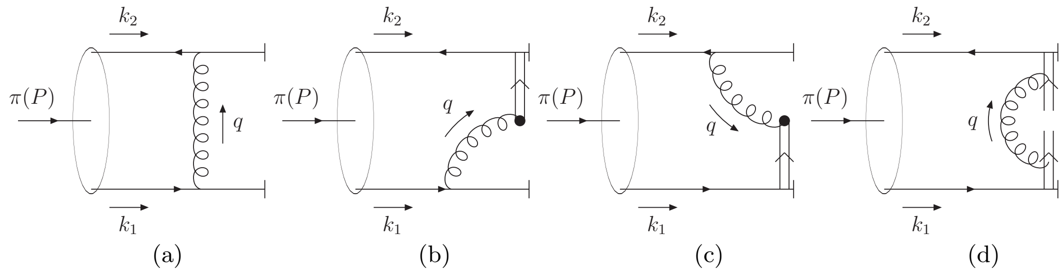

FIG. 2: NLO diagrams for the pion wave function with the loop momentum flowing through the hard kernel.

and the gluon mass λ_L is introduced to isolate the infrared (IR) pole. Applying the Feynman parametrization, it is easy to obtain

$$I = \frac{4\alpha\alpha_s}{\pi} \ln \frac{\lambda_L^2}{\mu^2} + \text{UV pole} , \tag{6}$$

where $\ln \lambda_L^2$ denotes the IR singularity [1], the explicit expression for the ultraviolet (UV) pole is irrelevant here, and μ is the renormalization scale. However, if employing the contour integration naively, Eq. (4) vanishes:

$$I = 16i\alpha g_s^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{(2q^+q^- - q_\perp^2 - \lambda_L^2 + i\varepsilon)[2q^+q^- - q_\perp^2 - (1-\alpha)\lambda_L^2 + i\varepsilon]} = 0 . \tag{7}$$

The poles in the above integrand are located in the same half plane, either upper or lower, no matter whether one integrates over the q^- or q^+ first. Therefore, one can close the contour in the other half plane not containing the poles. Obviously, Eq. (7) contradicts Eq. (6). How to understand this puzzle is the key to clarify the conflict between [1] and [6].

The above simple example illustrates that a naive application of the contour integration in the light-cone coordinates may lead to a false result. This is exactly the reason the authors of [1] drew the wrong conclusion from Fig. 2. When one closes the contour in Eq. (7) by including the semicircle at infinity, it has been implicitly assumed that the contribution from the semicircle is negligible. However, this assumption does not hold as $q^+ \to 0$ [7]: the product q^+q^- in the denominator does not provide a suppression on the semicircle of a large radius $|q^-|$ as $q^+ \to 0$. In other words, the poles $q^- = (q_\perp^2 + \lambda_L^2)/(2q^+)$ and $q^- = [q_\perp^2 + (1-\alpha)\lambda_L^2]/(2q^+)$, also moving to infinity as $q^+ \to 0$, cannot be avoided by the contour in Eq. (7). A safe way to proceed is to close the contour of q^- with a semicircle of a large but finite radius R in the half plane without the poles. For example, the semicircle in the upper half plane is considered for $q^+ > 0$ in Fig. 4(a). The integration over q^- along the real axis is thus equated to the integration over the semicircle, and Eq. (7) becomes

$$\begin{aligned} I &= 16i\alpha g_s^2 \lim_{R\to\infty} \int \frac{d^2q_\perp}{(2\pi)^4} \left[i\int_\pi^0 d\theta \int_0^\infty dq^+ + i\int_{-\pi}^0 d\theta \int_{-\infty}^0 dq^+ \right] \\ &\quad \times \frac{Re^{i\theta}}{(2q^+Re^{i\theta} - q_\perp^2 - \lambda_L^2 + i\varepsilon)[2q^+Re^{i\theta} - q_\perp^2 - (1-\alpha)\lambda_L^2 + i\varepsilon]} , \end{aligned} \tag{8}$$

where the first and second terms in the above square brackets correspond to Fig. 4(a) and Fig. 4(b), respectively. Performing the integration over q^+ and θ, and then taking the $R \to \infty$ limit, we reproduce Eq. (6). With the above

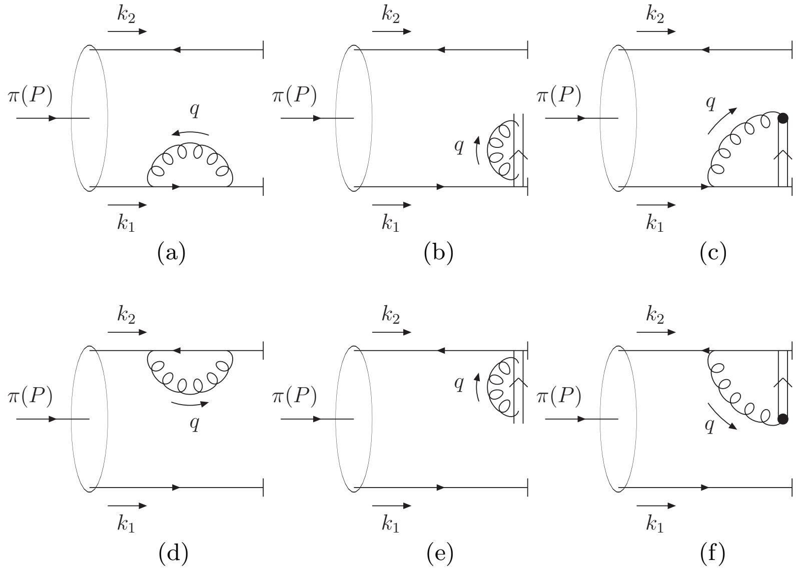

FIG. 3: NLO diagrams for the pion wave function without the loop momentum flowing through the hard kernel.

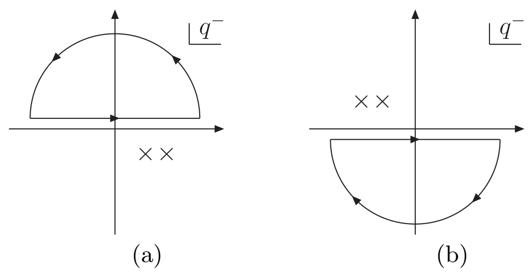

FIG. 4: Contours for Eqs. (7) and (8) in the regions (a) $q^+ > 0$ and (b) $q^+ < 0$.

prescription, the potential contribution from the large semicircles at $q^+ \to 0$ is taken into account, and a consistent result is obtained in the contour integration.

Below we recalculate Fig. 2 (the NLO diagrams for the pion wave function with the loop momentum flowing through the hard kernel) following the aforementioned method, and demonstrate that Figs. 2(a), 2(b), and 2(c) are in fact free of the gauge-dependent light-cone singularity $\alpha \ln \lambda_L^2$. We also recalculate Fig. 3 (the NLO diagrams for the pion wave function without the loop momentum flowing through the hard kernel), and point out the erroneous observation derived from Figs. 3(c) and 3(f) in [1]. Since the authors of [1] have evaluated the NLO diagrams for the form factor using the Feynman parametrization, these results are valid.

We start from Fig. 2(b), whose gauge-dependent part is written as [1]

$$\phi_\alpha|_{2b} = 16 i\alpha g_s^2 \int \frac{d^4q}{(2\pi)^4} \frac{2(k_1^+ - q^+)q^- - \vec{k}_{1\perp}\cdot\vec{q}_\perp + q_\perp^2}{[(k_1-q)^2 + i\varepsilon](q^2+i\varepsilon)^2} \delta(k^+ - (k_1^+ - q^+))\delta^2(\vec{k}_\perp - (\vec{k}_{1\perp} - \vec{q}_\perp)) \, . \tag{9}$$

The wave function $\phi_\alpha|_{2b}$ is convoluted with the LO hard kernel:

$$\phi_\alpha|_{2b} \otimes H^{(0)} = \int_0^1 dx \int d^2k_\perp \frac{1}{xQ^2 + k_\perp^2} \phi_\alpha|_{2b} \, , \tag{10}$$

with the variable $x \equiv k^+/P^+$. Integrating the two δ-functions over x and $k_\perp$, the LO hard kernel in Eq. (3) appears. In the light-cone region the scaling law for the components of q is defined by $(q^+, q^-, q_\perp) \sim (\delta^2, 1, \delta)$ with δ being a small parameter [1], according to which the first term $(k_1^+ - q^+)q^-$ in Eq. (9) gives the leading contribution. The light-cone singularity is regularized by introducing the gluon mass λ_L^2 as shown in Eq. (5). Applying the contour

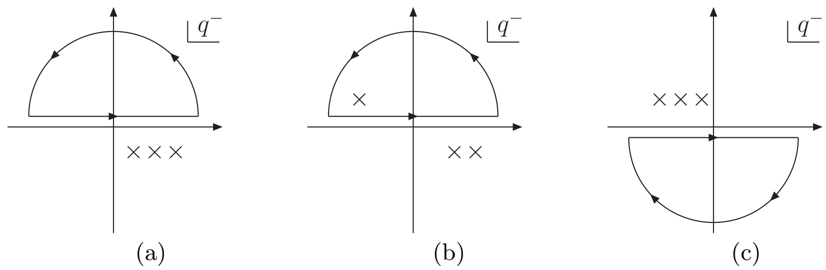

FIG. 5: Contours for Eqs. (9) and (12) in the regions (a) $q^+ > k_1^+$, (b) $0 < q^+ < k_1^+$, and (c) $q^+ < 0$.

integration over q^- for $0 < q^+ < k_1^+$ naively, we reproduce the result in [1],

$$\phi_\alpha^{\rm FMW}|_{2b} \otimes H^{(0)} = \frac{4\alpha\alpha_s}{P^+\pi}\frac{\ln\lambda_L^2}{x_0Q^2+k_{1\perp}^2} + \text{finite terms}\,. \tag{11}$$

As explained above, we have to examine the potential contribution from the large semicircles. For the range $q^+ > k_1^+$, all the poles are located in the lower half plane, so we close the contour of q^- with a large semicircle in the upper half plane shown in Fig. 5(a). The contribution from this semicircle remains negligible in the limit $q^+ \to k_1^+$, because of the suppression factor $(k_1^+ - q^+)$ in the numerator of Eq. (9). For $q^+ < 0$, all the poles are located in the upper half plane, so we close the contour with a large semicircle in the lower plane shown in Fig. 5(c). In this case, the semicircle may contribute as $q^+ \to 0$, and we equate the integral over q^- to the contribution from the semicircle of a radius R. For $0 < q^+ < k_1^+$, the single pole $q^- = (|\vec{k}_{1\perp} - \vec{q}_\perp|^2 - i\varepsilon)/[2(q^+ - k_1^+)]$ is located in the upper half plane and the double pole $q^- = (q_\perp^2 - i\varepsilon)/(2q^+)$ in the lower half plane. We close the contour of q^- through the upper half plane with a semicircle shown in Fig. 5(b), which picks up the residue from the single pole, giving Eq. (11). The contribution from the semicircle of a radius R needs to be subtracted, since it does not vanish at infinity as $q^+ \to 0$.

Therefore, Fig. 2(b), as convoluted with the LO hard kernel, contains the additional contribution from the two semicircles, one for $0 < q^+ < k_1^+$ and another for $q^+ < 0$,

$$\begin{aligned}\phi'_\alpha|_{2b} \otimes H^{(0)} &= \frac{16i\alpha g_s^2}{P^+}\lim_{R\to\infty}\int\frac{d^2q_\perp}{(2\pi)^4}\left[i\int_\pi^0 d\theta\int_0^{k_1^+}dq^+ + i\int_{-\pi}^0 d\theta\int_{-\infty}^0 dq^+\right] \\ &\times\frac{2(k_1^+-q^+)(Re^{i\theta})^2}{(2q^+Re^{i\theta}-q_\perp^2-\lambda_L^2)[2q^+Re^{i\theta}-q_\perp^2-(1-\alpha)\lambda_L^2][2(q^+-k_1^+)Re^{i\theta}-|\vec{k}_{1\perp}-\vec{q}_\perp|^2]} \\ &\times\frac{1}{(k_1^+-q^+)Q^2/P^+ + |\vec{k}_{1\perp}-\vec{q}_\perp|^2}\,.\end{aligned} \tag{12}$$

Working out the integration over q^+, θ and $q_\perp$, and then taking the $R \to \infty$ limit, we obtain

$$\phi'_\alpha|_{2b} \otimes H^{(0)} = -\frac{4\alpha\alpha_s}{P^+\pi}\frac{\ln\lambda_L^2}{x_0Q^2+k_{1\perp}^2} + \text{finite terms}\,, \tag{13}$$

which cancels the IR pole in Eq. (11) exactly, implying that Fig. 2(b) is free of the gauge-dependent light-cone singularity. Note that the limit $R \to \infty$ must be taken after the q^+ integration. Then the contribution from the semicircles at $q^+ = 0$ can be evaluated correctly. The same conclusion applies to Fig. 2(c).

We have explained the absence of the light-cone singularity in Fig. 2(b) through the contour integration over q^- by including the semicircle contribution. Each of the residue contribution in Eq. (11) and the semicircle contribution in Eq. (13) has the light-cone singularity, but they cancel each other exactly. It is thus understood why the authors in [1] got a fake light-cone singularity from Fig. 2(b): they have missed the contribution from the large semicircles. Their wrong result in Eq. (11) has an origin similar to that of Eq. (7). Performing the integration over q^+ first, we obtain the same result as in the case starting with the q^- integration. In the contour integration over q^+, the semicircle contribution must be handled appropriately as $q^- \to 0$. For the range $q^- > 0$, we equate the integral over q^+ to the contribution from the semicircle of a radius R in the upper half plane. For $q^- < 0$, we close the contour of q^+ through the lower half plane with a semicircle of a radius R, which picks up the residue from the pole $q^+ = (k_1^+Q^2 + P^+|\vec{k}_{1\perp} - \vec{q}_\perp|^2 - i\varepsilon)/Q^2$. It is found that neither the residue contribution nor the semicircle contribution contains the light-cone singularity, because the integral in Eq. (9) may be singular in the region $(q^+, q^-, q_\perp) \sim (\delta^2, 1, \delta)$, but not as $q^- \to 0$.

If applying the contour integration naively to Fig. 3(c),

$$\phi_\alpha|_{3c}\otimes H^{(0)}=-\frac{16i\alpha g_s^2}{P^+}\int\frac{d^4q}{(2\pi)^4}\frac{2(k_1^+-q^+)q^- -\vec{k}_{1\perp}\cdot\vec{q}_\perp+q_\perp^2}{[(k_1-q)^2+i\varepsilon](q^2+i\varepsilon)^2(x_0Q^2+k_{1\perp}^2)}\;, \tag{14}$$

the mistake made in [1] is more obvious. The term $(k_1^+-q^+)q^-$ gives a light-cone singularity from the region $(q^+,q^-,q_\perp)\sim(\delta^2,1,\delta)$ in the naive contour integration, which is the same as in Eq. (11). Since the loop momentum does not flow through the hard kernel, the term $(k_1^+-q^+)q^-$ also generates a UV divergence from the region $(q^+,q^-,q_\perp)\sim(1,\Lambda^2,\Lambda)$ with the scale $\Lambda\to\infty$ in the naive contour integration. However, according to the covariance argument [1], the corresponding loop integral, proportional to q^-, should vanish like $k_1^-=0$. To overcome this apparent contradiction, the authors in [1] dropped the gluon mass λ_L, adopted the dimensional regularization, misinterpreted the UV divergence as another light-cone singularity, and made the UV divergence and the light-cone singularity cancel each other. The momentum configuration $q\sim(1,\Lambda^2,\Lambda)$ should give a UV divergence, since it must be regularized with the number of dimensions $n<4$ in the dimensional regularization, and cannot be regularized by the gluon mass λ_L. Hence, we have difficulty to understand their argument [1] that a gluon with infinite invariant mass $q^2\sim\Lambda^2\to\infty$ produces the light-cone (infrared) singularity. The fact is that Fig. 3(c) has neither UV divergence nor light-cone singularity, after carefully including the contribution from the semicircles.

Our result is natural from the viewpoint of the Ward identity. Contracting the loop momentum q to the vertex on the internal quark line (see the numerator of the gauge-dependent term in Eq. (5)),

$$\frac{\not p-\not k_1}{(p-k_1)^2}\not q\frac{\not p-\not k_1+\not q}{(p-k_1+q)^2}=\frac{\not p-\not k_1}{(p-k_1)^2}-\frac{\not p-\not k_1+\not q}{(p-k_1+q)^2}\;, \tag{15}$$

the first (second) term leads to the LO hard kernel associated with Fig. 3(c) (Fig. 2(b)). The diagram Fig. 4c in [1] for the form factor is then factorized into convolutions of the LO hard kernel with Figs. 2(b) and 3(c). If Fig. 4c in [1] and Fig. 3(c) do not contain the light-cone singularity as observed in [1], Fig. 2(b) should not either. Another support to our result comes from the following simple observation. In the light-cone region the q^+ and $q_\perp$ dependence in the LO hard kernel is negligible. Then Figs. 2(b) and 3(c) involve exactly the same loop integral, namely, the same behavior in the light-cone region. If Fig. 3(c) does not produce the light-cone singularity as found in [1], Fig. 2(b) should not either.

We then turn to Fig. 2(a), which appears as a consequence of factorizing the box diagram Fig. 4a of [1] for the form factor. Again, Fig. 2(a) should not contain the gauge-dependent light-cone singularity, because Fig. 4a of [1] does not. The explicit expression for the leading contribution from Fig. 2(a) in the light-cone region is given by

$$\begin{aligned}\phi_\alpha|_{2a} &= -64i\alpha g_s^2\int\frac{d^4q}{(2\pi)^4}\frac{(k_1^+-q^+)(k_2^++q^+)(q^-)^2}{[(k_1-q)^2+i\varepsilon][(k_2+q)^2+i\varepsilon](q^2+i\varepsilon)^2}\\ &\qquad\times\delta(k^+-(k_1^+-q^+))\delta^2(\vec{k}_\perp-(\vec{k}_{1\perp}-\vec{q}_\perp))\;,\end{aligned} \tag{16}$$

where the numerator $(q^-)^2$ arises from $q^\mu q^\nu$ in the gluon propagator for $\mu=\nu=-$. Integrating over q^- first, we have two poles, $q^-=(|\vec{k}_{1\perp}-\vec{q}_\perp|^2-i\varepsilon)/[2(q^+-k_1^+)]$ for $0<q^+<k_1^+$ and $q^-=(|\vec{k}_{1\perp}-\vec{q}_\perp|^2-i\varepsilon)/[2(q^++k_2^+)]$ for $-k_2^+<q^+<0$. Closing the contour in the upper half plane for the former, and in the lower half plane for the latter naively, the result in [1] is reproduced,

$$\phi_\alpha^{\rm FMW}|_{2a}\otimes H^{(0)} = -\frac{4\alpha\alpha_s}{P^+\pi}\frac{\ln\lambda_L^2}{x_0Q^2+k_{1\perp}^2}+\text{finite terms}\;, \tag{17}$$

which is also a fake singularity.

Similarly, the contribution from the two semicircles of a radius R, which were added to form the closed contours mentioned above, is written as

$$\begin{aligned}\phi'_\alpha|_{2a}\otimes H^{(0)} &= -\frac{256i\pi\alpha\alpha_s}{P^+}\lim_{R\to\infty}\int\frac{d^2q_\perp}{(2\pi)^4}\left[i\int_\pi^0 d\theta\int_0^{k_1^+}dq^+ + i\int_{-\pi}^0 d\theta\int_{-k_2^+}^0 dq^+\right]\\ &\times\frac{(k_1^+-q^+)(k_2^++q^+)(Re^{i\theta})^3}{\left[-2(k_1^+-q^+)Re^{i\theta}-|\vec{k}_{1\perp}-\vec{q}_\perp|^2+i\varepsilon\right]\left[2(k_2^++q^+)Re^{i\theta}-|\vec{k}_{1\perp}-\vec{q}_\perp|^2+i\varepsilon\right]}\\ &\times\frac{1}{(2q^+Re^{i\theta}-q_\perp^2-\lambda_L^2+i\varepsilon)(2q^+Re^{i\theta}-q_\perp^2-(1-\alpha)\lambda_L^2+i\varepsilon)}\\ &\times\frac{1}{(k_1^+-q^+)Q^2/P^++|\vec{k}_{1\perp}-\vec{q}_\perp|^2-i\varepsilon}\;.\end{aligned} \tag{18}$$

The rest of the procedure is straightforward, leading to

$$\phi'_\alpha|_{2a}\otimes H^{(0)}=\frac{4\alpha\alpha_s}{P^+\pi}\frac{\ln\lambda_L^2}{x_0Q^2+k_{1\perp}^2}+\text{finite terms}\;. \tag{19}$$

The sum of Eqs. (17) and (19), i.e., Fig. 2(a), is free of the gauge-dependent light-cone singularity, contrary to the observation in [1]. In summary, all the diagrams in Figs. 2 and 3, except those with the gluons attaching only the Wilson lines (Figs. 2(d), 3(b), and 3(e)), do not generate the gauge-dependent IR singularity.

We then comment on the diagrams Figs. 2(d), 3(b), and 3(e). These diagrams do not appear in the factorization of collinear gluons [8, 9, 10] from the pion transition form factor. It has been shown that their sum is IR finite [1]. This must be the case, since all the IR divergences in the form factor diagrams, which arise from the collinear region with the scaling law $(q^+,q^-,q_\perp)\sim(1,\delta^2,\delta)$, have been absorbed into the other diagrams in Figs. 2 and 3 [6]. However, in view of a gauge invariant definition for the pion wave function, Figs. 2(d), 3(b), and 3(e) should be included. A resolution is to invoke a soft subtraction factor in the denominator [1, 8]:

$$\phi(x_0;x,k_\perp)=\int\frac{dy^-}{2\pi i}\frac{d^2y_\perp}{(2\pi)^2}e^{-i(1-x)P^+y^--i\vec{k}_\perp\cdot\vec{y}_\perp}\frac{\langle 0|\bar{q}(y)W_y(n)^\dagger I_{n;y,0}W_0(n)\gamma^+\gamma_5 q(0)|q(k_1)\bar{q}(k_2)\rangle}{\langle 0|W_y(n)^\dagger W_y(u)I_{n;y,0}I^\dagger_{u;y,0}W_0(n)W_0(u)^\dagger|0\rangle}\;, \tag{20}$$

which removes Figs. 2(d), 3(b), and 3(e) in a gauge invariant way. In the above definition, $y=(0,y^-,\vec{y}_\perp)$ is the coordinate of the anti-quark field $\bar{q}$, n with $n^2\neq 0$ the direction of the Wilson line, and $|q(k_1)\bar{q}(k_2)\rangle$ the leading Fock state of the pion. The factor $W_y(n)$ denotes the Wilson line operator

$$W_y(n)=P\exp\left[-ig_s\int_0^\infty d\lambda n\cdot A(y+\lambda n)\right]\;. \tag{21}$$

The two Wilson lines $W_y(n)$ and $W_0(n)$ must be connected by a link $I_{n;y,0}$ at infinity [10, 11].

The subtraction factor in the denominator generates the soft diagrams similar to Figs. 2 and 3, but with the fermion lines being replaced by the Wilson lines in the direction u. Besides Figs. 2(d), 3(b), and 3(e), additional soft diagrams, such as the vertex corrections with the gluons attaching the Wilson lines in the directions n and u, are introduced at the same time. To ensure that the soft subtraction does not change the collinear structure of the numerator, u should not lie on the light cone, namely, $u^2\neq 0$. Other than this requirement, the direction of u is completely arbitrary. Hence, the subtraction of Figs. 2(d), 3(b), and 3(e) will result in an arbitrary UV pole for the pion wave function. One can then take advantage of this arbitrariness, adjusting u so that the UV pole of the above vertex corrections cancels the UV pole of the self-energy corrections to the Wilson lines along u. Below we demonstrate this cancellation explicitly. Because the subtraction factor is gauge invariant, we evaluate the soft diagrams in the Feynman gauge, i.e., consider only the $g^{\mu\nu}$ tensor for the gluon propagator. Then the former is given by [12]

$$-\frac{\alpha_sC_F}{4\pi}\frac{u\cdot n}{\sqrt{(u\cdot n)^2-u^2n^2}}\ln\frac{\sqrt{(u\cdot n)^2-u^2n^2}+u\cdot n}{\sqrt{(u\cdot n)^2-u^2n^2}-u\cdot n}\left(\frac{1}{\epsilon}+\ln\frac{4\pi\mu^2}{\lambda_L^2e^{\gamma_E}}\right)\;, \tag{22}$$

and the latter by

$$\frac{\alpha_sC_F}{4\pi}\left(\frac{1}{\epsilon}+\ln\frac{4\pi\mu^2}{\lambda_L^2e^{\gamma_E}}\right)\;. \tag{23}$$

It is easy to find that the above two expressions cancel exactly for $u^2n^2<0$ and $u\cdot n/\sqrt{|u^2n^2|}\approx 0.85$. It is also trivial to show that the remaining soft diagrams with the loop momentum flowing through the hard kernel cancel, when the above conditions for u and n are satisfied. Eventually, we just need to calculate the seven effective diagrams in Fig. 2 of [6], i.e., Figs. 2(a)-2(c), and Figs. 3(a), 3(c), 3(d) and 3(f) in this comment, in the Feynman gauge at one-loop level.

After clarifying the conflict, the all-order proof of the leading-power k_T factorization theorem for the pion transition form factor in Sec. III of [6] follows straightforwardly. Below we mention the points of the proof briefly. Since the light-cone singularity does not exist, the substitution for the metric tensor of the gluon propagator in the covariant gauge, Eq. (47) of [6], works for extracting all IR (collinear) divergences. We then employ the derivative in Eq. (50) of [6] to collect the gauge-dependent terms arising from the off-shell partons. The derivative of the soft subtraction factor vanishes, since it is gauge invariant. The derivative of the pion wave function is then related to the derivative of the numerator in Eq. (20), which, after applying the Ward identity, gives Eq. (53) in [6]. Combining Eqs. (52) and (53) in [6], the k_T factorization theorem for the pion transition form factor is proved by induction.

In conclusion, the only necessary revision for [6] is to replace the definition for the k_T-dependent pion wave function by Eq. (20) with an appropriate vector u. Except this modification, all the results in [6] are valid. The gauge-dependent light-cone singularity found in [1] does not exist; their result is attributed to a careless application of

the contour integration in the light-cone coordinates. The correct prescription is to keep the contribution from the semicircles of a finite radius first, work out the loop integration, and then move the semicircles to infinity. We have confirmed that the all-order proof of the k_T factorization theorem for the pion transition form factor holds: the gauge dependence, arising from the off-shell external partons, cancels between the full QCD diagrams for the form factor and the effective diagrams for the pion wave function. Therefore, the k_T factorization produces a gauge invariant and IR finite hard kernel, and provides a solid basis for the PQCD approach to exclusive B meson decays.

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